10.7

Solution. The poles of the z-transform are

$$z_1 = \frac{1}{2}j, \qquad z_2 = -\frac{1}{2}j, \qquad z_3 = \frac{3}{4}, \qquad z_4 = -\frac{1}{2}.$$

Based on these poles, the ROC can be

$$0 < \|z\| < \frac{1}{2}, \qquad \frac{1}{2} < \|z\| < \frac{3}{4}, \qquad \text{or } \frac{3}{4} < \|z\|.$$

10.16

Note that, for a stable discrete-time LTI system with rational system function $H(z)$, the condition (a) for causality is equivalent to "the outermost pole is inside the unit circle".

(a)

Solution. The given z-transform has a pole at infinity. Therefore, it is not causal.

(b)

Solution. The poles of the z-transforklkjm are at $z = \frac{1}{4}$ and $z = -\frac{3}{4}$. Therefore, it is causal.

(c)

Solution. The given z-transform has a pole at $z = \frac{4}{3}$. Therefore, it is not causal.

10.21

(a)

Solution. For $x[n] = \delta[n+5]$,

$$X(z) = z^5, \qquad \text{All } z,$$

as shown in the figure below.

TODO

(g)

Solution. For $x[n] = 2^n u[-n] + \frac{1}{4^n} u[n-1]$, by table 10.1 and 10.2,

$$X(z) = \frac{2}{2-z} + \frac{1}{4z-1}, \qquad \frac{1}{4} < \|z\| < 2,$$

as shown in the figure below.

TODO

10.24

(a)

Solution. We have

$$X(z) = \frac{1 - 2z^{-1}}{\left(1 + \frac{1}{2}z^{-1}\right)(1 + 2z^{-1})} = \frac{8}{3(1 + 2z^{-1})} - \frac{5}{3\left(1 + \frac{1}{2}z^{-1}\right)}.$$

Since $x[n]$ is absolutely summable, the ROC is $\frac{1}{2} < \|z\| < 2$. It follows that

$$x[n] = \frac{8}{3}\left(-\frac{1}{2}\right)^n u[n] - \frac{5}{3}(-2)^n u[n].$$

(b)

Solution. We have

$$X(z) = 1 - z^{-1} + \frac{1}{2}z^{-2} - \frac{1}{4}z^{-3} + \cdots = 1 - \sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^{n-1} z^{-n}.$$

Using the analysis equation, we get

$$x[n] = \delta[n] - \left(-\frac{1}{2}\right)^{n-1} u[n-1].$$

10.33

(a)

Solution. Taking the z-transform of the equation and simplifying, we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}}.$$

(b)

Solution. If $x[n] = \left(\frac{1}{2}\right)^n u[n]$, then

$$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}, \qquad \|z\| > \frac{1}{2}.$$

Hence

$$Y(z) = H(z)X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{\frac{1}{2}z^{-1}}{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}}.$$

Using table 10.2, we get

$$y[n] = \left(\frac{1}{2}\right)^n u[n] + \frac{2}{\sqrt{3}}\left(\frac{1}{2}\right)^n \sin\left(\frac{\pi}{3}n\right) u[n].$$

10.37

(a)

Solution. Let $w[n]$ denote the signal represented by the top-middle node. Then from the diagram we know that

$$x[n] = w[n] + \frac{1}{3}w[n-1] - \frac{2}{9}w[n-2],$$
$$y[n] = w[n] - \frac{9}{8}w[n-1].$$

Hence

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{Y(z)}{W(z)}}{\frac{X(z)}{W(z)}} = \frac{1 - \frac{9}{8}z^{-1}}{1 + \frac{1}{3}z^{-1} - \frac{2}{9}z^{-2}}.$$

Taking the inverse z-transform, we obtain

$$y[n] + \frac{1}{3}y[n-1] - \frac{2}{9}y[n-2] = x[n] - \frac{9}{8}x[n-1].$$

(b)

Solution. The poles of the z-transform are at $z = \frac{1}{3}$ and $z = -\frac{2}{3}$. Therefore, the system is stable.

10.42

(b)

Solution. Applying the unilateral z-transform, we obtain

$$\mathcal{Y}(z) - \frac{1}{2}z^{-1}\mathcal{Y}(z) - \frac{1}{2}y[-1] = \mathcal{X}(z) - \frac{1}{2}z^{-1}\mathcal{X}(z).$$

For the zero-input response, setting $\mathcal{X}(z) = 0$, we get $\mathcal{Y}(z) = 0$. Then by applying the inverse unilateral z-transform, we obtain $y_{\text{zi}}[n] = 0$.

For the zero-state response, setting $y[-1] = 0$, we get $\mathcal{Y}(z) = \mathcal{X}(z) = \frac{1}{1-z^{-1}}$. Then by applying the inverse unilateral z-transform, we obtain $y_{\text{zs}}[n] = u[n]$.

10.59

(a)

Solution. Let $w[n]$ denote the signal represented by the top-middle node. Then from the diagram we know that

$$x[n] = w[n] + \frac{k}{3}w[n-1],$$
$$y[n] = w[n] - \frac{k}{4}w[n-1].$$

Hence

$$H(z)=\frac{Y(z)}{X(z)}=\frac{\frac{Y(z)}{W(z)}}{\frac{X(z)}{W(z)}}=\frac{1-\frac{k}{4}z^{-1}}{1+\frac{k}{3}z^{-1}}.$$

Since $H(z)$ correspond to a causal filter, the ROC is $\frac{\|k\|}{3}<\|z\|$, as shown in the figure below.

TODO

(b)

Solution. For the system to be stable, there must be $\frac{\|k\|}{3}<1$, that is, $\|k\|<3$.

(c)

Solution. Since $x[n]=\left(\frac{2}{3}\right)^n$, we have

$$y[n]=H\left(\frac{2}{3}\right)x[n]=\frac{1-\frac{1}{4}\left(\frac{2}{3}\right)^{-1}}{1+\frac{1}{3}\left(\frac{2}{3}\right)^{-1}}\left(\frac{2}{3}\right)^n=\frac{5}{12}\left(\frac{2}{3}\right)^n.$$