9.21

(b)

Solution. Using Euler's relation, we can write

$$x(t) = \left(e^{-4t} - \frac{j}{2}e^{-(5-5j)t} + \frac{j}{2}e^{-(5+5j)t}\right)u(t).$$

Then the Laplace transform of $x(t)$ can be expressed as

$$X(s) = \int_{-\infty}^{\infty} e^{-4t}u(t)\,\mathrm{d}t - \frac{j}{2}\int_{-\infty}^{\infty} e^{-(5-5j)t}u(t)\,\mathrm{d}t + \frac{j}{2}\int_{-\infty}^{\infty} e^{-(5+5j)t}u(t)\,\mathrm{d}t.$$

Each of these integrals represents a Laplace transform of the type encountered in Example 9.1. It follows that

$$\begin{aligned}
e^{4t}u(t) &\overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+4}, &\qquad \mathcal{Re}\{s\} > -4,\\
e^{-(5-5j)t}u(t) &\overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s-5+5j}, &\qquad \mathcal{Re}\{s\} > -5,\\
e^{-(5+5j)t}u(t) &\overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s-5-5j}, &\qquad \mathcal{Re}\{s\} > -5.
\end{aligned}$$

For all three Laplace transforms to converge simultaneously, we must have $\mathcal{Re}\{s\} > -4$. Consequently, the Laplace transform of $x(t)$ is

$$X(s) = \frac{1}{s+4} - \frac{j}{2(s-5+5j)} + \frac{j}{2(s-5-5j)}, \qquad \mathcal{Re}\{s\} > -4.$$

(i)

Solution.

$$x(t) = \delta(t) + u(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s) = 1 + \frac{1}{s}, \qquad \mathcal{Re}\{s\} > 0.$$

(j)

Solution. Note that $\delta(3t) + u(3t) = \delta(t) + u(t)$. Therefore, the Laplace transform is the same as the result of the previous part.

9.22

(e)

Solution. Let

$$X(s) = \frac{s+1}{s^2+5s+6} = \frac{2}{s+3} - \frac{1}{s+2}.$$

From the given ROC, we know that $x(t)$ must be a two-sided signal. Therefore,

$$x(t) = 2e^{-3t}u(t) + e^{-2t}u(-t), \qquad \mathcal{Re}\{s\} > -2.$$

9.23

The four pole-zero plots shown may have the following possible ROCs:

- Plot 1: $\mathcal{Re}\{s\} < -2$ or $-2 < \mathcal{Re}\{s\} < 2$ or $2 < \mathcal{Re}\{s\}$.
- Plot 2: $\mathcal{Re}\{s\} < -2$ or $-2 < \mathcal{Re}\{s\}$.
- Plot 3: $\mathcal{Re}\{s\} < 2$ or $2 < \mathcal{Re}\{s\}$.
- Plot 4: The entire s-plane.

Let R denote the ROC of the Laplace transform $X(s)$ of the signal $x(t)$.

(1)

Solution. From table 9.1, we know that

$$x(t)e^{-3t} \overset{\mathcal{L}}{\longleftrightarrow} X(s+3).$$

The ROC R_1 of this new Laplace transform is R shifted to the left by 3. Since $x(t)e^{-3t}$ is absolutely integrable, R_1 must contain the $j\omega$ axis.

- For plot 1, this is possible only if R was $2 < \mathcal{Re}\{s\}$.
- For plot 2, this is possible only if R was $-2 < \mathcal{Re}\{s\}$.
- For plot 3, this is possible only if R was $2 < \mathcal{Re}\{s\}$.
- For plot 4, R is the entire s-plane.

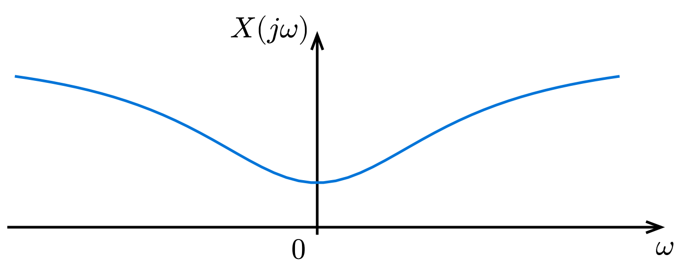
9.25

(c)

Solution. Let α and β denote the pole and zero of $X(s)$, respectively. Then

$$\|X(j\omega)\| = M\sqrt{\frac{\omega^2 + \beta^2}{\omega^2 + \alpha^2}},$$

as shown in the figure below.



9.26

Solution. From table 9.1, we know that

$$Y(s) = e^{-2s}X_1(s) \cdot e^{-3s}X_2(-s) = \frac{e^{-5s}}{6+s-s^2}.$$

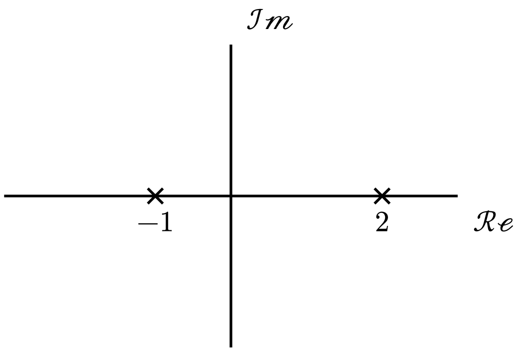
9.31

(a)

Solution. By taking the Laplace transform and simplifying, we obtain

$$H(s) = \frac{1}{s^2 - s - 2}.$$

The pole-zero plot of $H(s)$ is shown in the figure below.



(b)

Solution. The partial fraction expansion of $H(s)$ is

$$H(s) = \frac{1}{3(s-2)} - \frac{1}{3(s+1)}.$$

1. If the system is stable, then the ROC has to be $-1 < \mathcal{R}e\{s\} < 2$. Therefore,

$$h(t) = -\frac{1}{3}e^{2t}u(-t) - \frac{1}{3}e^{-t}u(t).$$

2. If the system is causal, then the ROC has to be $2 < \mathcal{R}e\{s\}$. Therefore,

$$h(t) = \frac{1}{3}e^{2t}u(t) - \frac{1}{3}e^{-t}u(t).$$

3. If the system is neither stable nor causal, then the ROC has to be $\mathcal{R}e\{s\} < -1$. Therefore,

$$h(t) = -\frac{1}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(-t).$$

9.35

(a)

Solution. Let $w(t)$ denote the signal represented by the bottom-middle node. Then the diagram shows that

$$
\begin{aligned}
\frac{\mathrm{d}^2 w(t)}{\mathrm{d}t^2} + 2\frac{\mathrm{d}w(t)}{\mathrm{d}t} + w(t) &= x(t), \\
\frac{\mathrm{d}^2 w(t)}{\mathrm{d}t^2} - \frac{\mathrm{d}w(t)}{\mathrm{d}t} - 6w(t) &= y(t).
\end{aligned}
$$

Hence

$$
H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{Y(s)}{W(s)}}{\frac{X(s)}{W(s)}} = \frac{s^2 - s - 6}{s^2 + 2s + 1}.
$$

Taking the inverse Laplace transform, we obtain

$$
\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = \frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2} - \frac{\mathrm{d}x(t)}{\mathrm{d}t} - 6x(t).
$$

(b)

Solution. From the previous result, the two poles of the system are at -1. Since the system is causal, the ROC is $-1 < \mathcal{Re}\{s\}$ and hence includes the $j\omega$-axis. Therefore, the system is stable.

9.40

Taking the unilateral Laplace transform of the equation, we obtain

$$
\begin{gathered}
s^3\mathcal{Y}(s) - s^2 y(0^-) - s y'(0^-) - y''(0^-) + \\
6s^2\mathcal{Y}(s) - 6sy(0^-) - 6y'(0^-) + 11s\mathcal{Y}(s) - 11y(0^-) + 6\mathcal{Y}(s) = \mathcal{X}(s).
\end{gathered}
$$

(a)

Solution. For the zero-state response, we have

$$
s^3\mathcal{Y}(s) + 6s^2\mathcal{Y}(s) + 11s\mathcal{Y}(s) + 6\mathcal{Y}(s) = \mathcal{X}(s) = \frac{1}{s+4}.
$$

Therefore,

$$
\mathcal{Y}(s) = \frac{1}{(s+4)(s^3 + 6s^2 + 11s + 6)} = \frac{1}{2(s+2)} - \frac{1}{2(s+3)} - \frac{1}{6(s+1)} + \frac{1}{6(s+4)}.
$$

Taking the inverse unilateral Laplace transform, we obtain

$$
y(t) = \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}e^{-3t}u(t) - \frac{1}{6}e^{-t}u(t) + \frac{1}{6}e^{-4t}u(t).
$$

(b)

Solution. For the zero-input response, with the given initial condition, we can obtain

$$
\mathcal{Y}(s) = \frac{s^2 + 5s + 6}{s^3 + 6s^2 + 11s + 6} = \frac{1}{s+1}.
$$

Taking the inverse unilateral Laplace transform, we obtain

$$y(t) = e^{-t}u(t).$$

(c)

Solution. The total response is the sum of the zero-state and zero-input responses. Therefore,

$$y(t) = \left(\frac{1}{2}e^{-2t} - \frac{1}{2}e^{-3t} + \frac{5}{6}e^{-t} + \frac{1}{6}e^{-4t}\right)u(t).$$